On the Convergence of Jacobi Series at the Poles

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Abstract

The most urgent need in information Technology is data compilation for different purposes in varying ways. Convergence of Jacobi series is used for replacement of discontinuous signals by its approximated absolutely continuous signals for data manipulation in computers. Here in this paper a Banach space X of signals which are p-power $(1 \le p \le \infty)$ Lebesgue integrable with weight

$$\omega(x) = (1-x)^{\alpha}(1+x)^{\beta}, (\alpha > -1, \beta > -1)$$

on [-1,1] is considered. Some of subspaces of X have been recognized by the convergence behavior of Fourier-Jacobi expansions associated with the signals. These results are applied to signal processing with wavelets related to useful concept in Science and Engineering disciplines.

Keywords

Fourior-Jacobi Expansion; Signal Processing; Data Repairs; Wavelets

Introduction

Convergency of an infinite series has always been a challenge to mathematicians. Convergence of Jacobi series not only leads to uniform convergence or an approximation over the interval [-1, 1] but explains transform of a signal into absolutely continuous signal. On and on this has potential applications such as for contaminated noise removal, corrupted data repairs and many more in information technology in the Science and Engineering branches. Let X denote either the space C[-1,1] of all continuous signals or the space of p-power Lebesgue integrable signals with weight $w(x) = (1-x)^{\alpha}(1+x)^{\beta};\ (\alpha, \beta > -1)$ on [-1,1]. Sup and p-norms are defined as usual. A series called Fourier-Jacobi expansion is associated (see Szegö [3], Chapter IX) with every $f \in X$ as

$$f(\cos\theta) \sim \sum_{n=0}^{\infty} a_n P_n^{(\alpha,\beta)}(\cos\theta) \equiv$$

$$\sum_{n=0}^{\infty} f(n)\, \omega_n^{(\alpha,\beta)} R_n^{(\alpha,\beta)}(\cos\theta) \tag{1.1}$$

where $f(n)$ is nth Fourier-Jacobi Transform of f such that

$$f(n) = \int_0^{\pi} f(\cos\theta) R_n^{(\alpha,\beta)}(\cos\theta) \rho^{(\alpha,\beta)}(\theta) d\theta \tag{1.2}$$

$$R_n^{(\alpha,\beta)}(\cos\theta) = \frac{P_n^{(\alpha,\beta)}(\cos\theta)}{P_n^{(\alpha,\beta)}(1)} \tag{1.3}$$

is orthonormalized Jacobi polynomial, $(n = 0, 1, 2, \ldots)$ where

$$\int_0^{\pi} R_n^{(\alpha,\beta)}(\cos\theta) R_m^{(\alpha,\beta)}(\cos\theta) \rho^{(\alpha,\beta)}(\theta) d\theta = \delta_{nm} \left\{\omega_n^{(\alpha,\beta)}\right\}^{-1} \tag{1.4}$$

$$\omega_n^{(\alpha,\beta)} = \frac{(2n+\alpha+\beta+1)\Gamma(n+\alpha+\beta+1)\Gamma(n+\alpha+1)}{\Gamma(n+\beta+1)\Gamma(n+1)\Gamma(\alpha+1)\Gamma(\alpha+1)}$$

$$= \frac{n^{2\alpha+1}}{\{\Gamma(\alpha+1)\}^2}[1 + O(1/n)] \equiv n^{2\alpha+1} L(n), (say); \tag{1.5}$$

and

$$\rho^{(\alpha,\beta)}(\theta) = \omega(\cos\theta)\sin\theta \equiv$$
$$2^{(\alpha+\beta+1)} (\sin\theta/2)^{2\alpha+1} (\cos\theta/2)^{2\beta+1} \tag{1.6}$$

$P_n^{(\alpha,\beta)}(\cos\theta)$ is the nth Jacobi polynomial of order (α, β) and degree n (see Szegö [3]). δ_{nm} is the Kronecker delta. To avoid confusions and have easy access to verifications of formulae, the notations of Szego are employed [3]. As consequence, we write

$$a_n = \left\{h_n^{(\alpha,\beta)}\right\}^{-1} \int_0^{\pi} f(\cos\theta) P_n^{(\alpha,\beta)}(\cos\theta) \rho^{(\alpha,\beta)}(\theta) d\theta \tag{1.7}$$

where

$$h_n^{(\alpha,\beta)} = \int_0^{\pi} [P_n^{(\alpha,\beta)}(\cos\theta)]^2 \rho^{(\alpha,\beta)}(\theta) d\theta$$
$$= \frac{1}{2n+\alpha+\beta+1} \frac{\Gamma(n+\alpha+1)\Gamma(n+\beta+1)}{\Gamma(n+1)\Gamma(n+\alpha+\beta+1)}$$
$$\lesssim O(1/n) \tag{1.8}$$

Hence the study of Jacobi series will make no

confusion when one uses orthonormalized Jacobi polynomials $R_n^{(\alpha,\beta)}(\cos\theta)$ in place of $P_n^{(\alpha,\beta)}(\cos\theta)$, divided by $P_n^{(\alpha,\beta)}(1)$.

Convergency of Jacobi Series at End Points

It is obvious that every signal of the space X can be associated to a unique Jacobi series given by (1.1) if the integrals in (1.7) exist for every $n = 0, 1, 2 ....$ Unique is in the sense that two signals are treated the same if they are equal almost everywhere in the interval of their definitions. It is natural to examine the circumstances in which this series (1.1) of Jacobi polynomials converges to the strength of the signal of which it is an outcome. The first result in this direction, known to us is that of Raü [2] where continuity of the signal f in the whole interval [-1, 1] is required and $-1 < \alpha < -1/2$ is must. Improving the results of (Szegö [3] Chapter IX theorem 9.1.4), Yadav, S. P. [4] has recognized the subspaces $X_i^{(\alpha,\beta)} \subset X, (i = 1,2,3,4 \text{ and } 5)$, where weaker than the continuity of f is required at the 'pole' and something extra at the other end $x = -1$. Even then these spaces exclude from the case $\alpha = -1/2$ recognized in Yadav [4]. Moreover, Szegö ([3](9.41.17) page 262 see the case k = 0) shows that continuous functions/signals on [-1,1] exist in X so that its associated Jacobi series diverges on $x = +1$ for $\alpha = -1/2$. It is natural to find the circumstances in which the case $\alpha = -1/2$ is covered. It is shown that the Jacobi series associated to signals which satisfy an integrability pole condition (2.1), converges to A at $x = +1$ for $-1 < \alpha \leq -1/2$ and $\beta > -1$. We assume

$$\int_0^t \varphi^{\alpha-1/2}\left|f(\cos\varphi) - A\right|d\varphi = o(t^{\alpha+1/2}) \tag{2.1}$$

as t → +0, where A is a constant depending on f only. This is a condition on f at $x = +1$. The condition (2.1) is independent of the continuity of f in [-1, 1] or at $x = +1$ but stronger than that assumed in Yadav [4]. The condition in Yadav [4] at $x = +1$ is

$$\int_0^t \varphi^{2\alpha+1}|f(\cos\varphi) - A|d\varphi = o\left(t^{2\alpha+2}\right) \tag{2.2}$$

as t → +0, where A is a constant depending on f only. For all $\alpha \geq -1/2$, (2.1) is implied (2.2) as

$$\int_0^t \varphi^{2\alpha+1}|f(\cos\varphi) - A|d\varphi$$
$$\leq t^{\alpha+3/2}\int_0^t \varphi^{\alpha-1/2}|f(\cos\varphi) - A|d\varphi$$
$$= o\left(t^{2\alpha+2}\right) \tag{2.3}$$

by (2.1) as t → +0. However, its converse is obviously not always possible. On the other hand continuity implies (2.2) but not (2.1). As $|f(\cos\varphi) - f(1)| = o(1)$ for $\varphi \to +0$ indicates that the integral in (2.1) diverges for $-1 < \alpha \leq -1/2$ while (2.2) holds. Moreover, the condition (2.1) is not trivial as functions of class $Lip\ \delta\ (\delta \geq 1/4)$ satisfy it. Let A = $f(1)$ then

$$\int_0^t \varphi^{\alpha-1/2}|f(\cos\varphi) - f(1)|d\varphi$$
$$\leq C_1\int_0^t \varphi^{\alpha-1/2}|1-\cos\varphi|^{\delta}d\varphi, (\text{as } f \in Lip\ \delta) \leq C_2\int_0^t \varphi^{\alpha-1/2+2\delta}\,d\varphi$$
$$\leq C_3\int_0^t \varphi^{\alpha}d\varphi, (\text{for } \delta \geq 1/4)$$
$$= O(t^{\alpha+1}) = o(t^{\alpha+1/2}) \tag{2.4}$$

as $t \to +0$, for $\alpha > -1$. $C_i (i = 1, 2, ...)$ are absolute constants, but not the same everywhere in this article until and unless stated otherwise. This shows that the class Lip δ satisfies (2.1). Lipschitz condition on f is necessary only in the arbitrarily small neighborhood of $x = +1$. Moreover, signals are expected to satisfy the condition (2.1). The conditions on $x = -1$ are called 'antipole' conditions e assumed as follows just the same as in Yadav [4].

$$\int_0^h \varphi^{\beta}|f(-\cos\varphi)|d\varphi = o(h^{\alpha}) \tag{2.5}$$

as $h \to +0$, for $\beta > -1$. In certain cases a condition lighter than (2.5) is assumed as

$$\int_0^s \varphi^{\beta+1/2}|f(-\cos\varphi)|d\varphi = o(s^{\alpha+1/2}) \tag{2.6}$$

as $s \to +0$, for $\beta > -1/2$. Following end point convergency results of Jacobi series are mile stones in the literature of Jacobi series as some results can be found on representing the signals in terms of Jacobi polynomials and wavelets. The pattern of notations is adopted in continuation of Yadav [4] and the followings are proved:

Theorem 1 Let $X_6^{\alpha,\beta} \subset X$ be a subspace of signals which satisfy the 'pole' condition (2.1) for $-1 < \alpha \leq -1/2$ and $-1 < \beta \leq -1/2$. Then the Jacobi series (1.1) associated to $f \in X_6^{\alpha,\beta}$ converges to A at the point $x = +1$.

Theorem 2 Let $X_7^{\alpha,\beta} \subset X$ be a subspace of signals which satisfy the 'pole' condition (2.1) for $-1 < \alpha \leq -1/2$

and $\beta > -1/2$ but $\alpha+\beta \leq -1$. Then the Jacobi series (1.1) associated to $f \in X_7^{\alpha,\beta}$ converges to A at the point $x=+1$.

To overcome the restriction of $\alpha+\beta \leq -1$, we need a lighter 'antipole' condition (2.6) so that, we have.

Theorem 3 Let $X_8^{\alpha,\beta} \subset X$ be a subspace of signals which satisfy the 'pole' condition (2.1) along with the 'antipole' condition (2.6) for $-1<\alpha\leq -1/2$ and $\beta > -1/2$ but $\alpha+\beta > -1$. Then the Jacobi series (1.1) associated to $f \in X_8^{\alpha,\beta}$ converges to A at the point $x=+1$. Improving the heaviness of the 'antipole' condition leads to the removal of the restriction from β to get the following:

Theorem 4 Let $X_9^{\alpha,\beta} \subset X$ be a subspace of signals which satisfy the 'pole' condition (2.1) along with the 'antipole' condition (2.5) for $-1<\alpha\leq -1/2$ and $\beta > -1$. Then the Jacobi series (1.1) associated to $f \in X_9^{\alpha,\beta}$ converges to A at the point $x=+1$.

All these spaces $X_i^{\alpha,\beta} \subset X\ (i=1,2,\dots,9)$ are normalized Banach subspaces as shown in Yadav [4]. Our theorems 1, 2, 3 and 4 hold good at the other end point $x=-1$, and only change of α,β is apparent and the 'antipole' condition should hold at $x=+1$.

Lemmas and Known Results to be Used

Following order estimates of the Jacobi polynomials are taken from Szegö [3] to prove our theorems.

Lemma 3.1 (Szegö [3] theorem (7.32.2). Let α,β be arbitrary and real and c a fixed positive constant, $n \to \infty$. Then

$$P_n^{(\alpha,\beta)}(\cos\theta) = \begin{cases} \theta^{-\alpha-1/2}O(n^{-1/2}), & c/n \leq \theta \leq \pi/2 \\ O(n^{\alpha}) & 0 \leq \theta \leq c/n \end{cases} \tag{3.1}$$

Moreover,

$$P_n^{(\alpha,\beta)}(-\cos\theta) = (-1)^n P_n^{(\beta,\alpha)}(\cos\theta) \text{ and}$$

$$P_n^{(\alpha,\beta)}(1) = \binom{n+\alpha}{n}$$

(see Szegö [3] page 59 and 68 respectively).

Lemma 3.2 (Szegö [3] theorems (8.21.8) and (8.21.13). Let α,β be arbitrary and real numbers, $n \to \infty$. Then

$$P_n^{(\alpha,\beta)}(\cos\theta) =$$

$$\begin{cases} n^{-1/2}k(\theta)\cos(n\theta+\gamma)+O(n^{-3/2}), & 0<\theta<\pi \\ n^{-1/2}k(\theta)\cos(n\theta+\gamma)+O(n\sin\theta)^{-1}O(1), & c/n\leq\theta\leq\pi-c/n \end{cases} \tag{3.2}$$

where

$$K(\theta) = \pi^{-1/2}(\sin\theta)^{-\alpha-1/2}(\cos\theta/2)^{-\beta-1/2}$$

and

$$N = n+(\alpha+\beta+1)/2;\quad \gamma = -(\alpha+1/2)\pi/2.$$

Lemma 3.3. (Szegö [3] page 71 (4.5.3)) For arbitrary α,β, we have

$$\sum_{i=0}^{n} h_i^{(\alpha,\beta)} P_i^{(\alpha,\beta)}(\cos\theta) P_i^{(\alpha,\beta)}(1)$$

$$= 2^{-(\alpha+\beta+1)} \frac{\Gamma(n+\alpha+\beta+2)}{\Gamma(\alpha+1)\Gamma(n+\beta+1)} P_n^{(\alpha+1,\beta)}(\cos\theta) \tag{3.3}$$

Lemma 3.4 (Askey and Wainger [1] page 470) For arbitrary α,β, we have

$$\sum_{i=0}^{n} h_i^{(\alpha,\beta)} P_i^{(\alpha,\beta)}(\cos\theta) P_i^{(\alpha,\beta)}(1)$$

$$= \frac{n+\alpha+\beta+1}{2n+\alpha+\beta+1} h_n^{(\alpha,\beta)} P_n^{(\alpha+1,\beta)}(\cos\theta) P_n^{(\alpha,\beta)}(1)$$

$$= h_n^{(\alpha+1,\beta)} P_n^{(\alpha+1,\beta)}(\cos\theta) P_n^{(\alpha+1,\beta)}(1) A_n \tag{3.4}$$

where $A_n = A_1 n^{-1} + A_2 n^{-2} + \dots + O(n^{-\lambda})$ for any $\lambda > 0$ and $A_1, A_2 \dots$ absolute contants.

Lemma 3.5 Let a signal $f \in X_i^{(\alpha,\beta)}$, $(i = 6,7,8 \text{ and } 9)$ satisfy (2.1) for $-1<\alpha\leq -1/2$ and β as in the respective spaces with the required 'antipole' condition. Then the following estimate.

$$\int_0^{\pi} F(\varphi) P_n^{(\alpha+1,\beta)}(\cos\varphi)\, d\varphi = o\left(n^{-\alpha-1}\right) \tag{3.5}$$

holds as $n \to \infty$, where

$$F(\varphi) = f(\cos\varphi)(\sin\varphi/2)^{2\alpha+1}(\cos\varphi/2)^{2\beta+1} \tag{3.6}$$

Proof: We break the integral in (3.5) as

$$\int_0^{c/n} + \int_{c/n}^{\delta} + \int_{\delta}^{\pi-\delta^1} + \int_{\pi-\delta^1}^{\pi-c/n} + \int_{\pi-c/n}^{\pi} \equiv \sum_{i=1}^{5} T_i \ (say).$$

where c, δ and δ^1 are arbitrarily small but fixed positive reals. The second order of Jacobi polynomial given for α,β arbitrary in (3.1) is use to get.

$$T_1 = O(n^{\alpha+1}) \int_0^{c/n} \varphi^{2\alpha+1}|f(\cos\varphi)-A|\,d\varphi$$

$$= O(n^{\alpha+1}n^{-\alpha-3/2}) \int_0^{c/n} \varphi^{\alpha-1/2}|f(\cos\varphi)-A|\,d\varphi$$

$$= o(n^{-\alpha-1}) \tag{3.7}$$

by the 'pole' condition (2.1) for $-1<\alpha\leq -1/2$. Again by the first equation of (3.1), we have

$$T_2 = O(n^{-1/2}) \int_{c/n}^{\delta} \varphi^{-\alpha-3/2}\varphi^{2\alpha+1}|f(\cos\varphi)-A|\,d\varphi$$

$$= O(n^{-1/2}) \int_{c/n}^{\delta} \varphi^{\alpha-1/2}|f(\cos\varphi)-A|\,d\varphi$$

But, let us write

$$G_1 = \int_0^t \varphi^{\alpha-1/2} |f(\cos\varphi) - A| d\varphi$$

and examine the meaning of the condition (2.1) which tells that

$$\forall \varepsilon > 0, \exists \delta > 0 : c/n < \varphi < \delta \Rightarrow |G_1(t)| < \varepsilon t^{\alpha+1/2}$$

Therefore,

$$\begin{aligned} T_2 &= O(n^{-1/2}) \left[\varepsilon\, t^{\alpha+1/2}\right]_{c/n}^{\delta} \\ &= O(n^{-1/2}) \left[\varepsilon\, \delta^{\alpha+1/2} - \varepsilon\, (c/n)^{\alpha+1/2}\right] \\ &= o(n^{-\alpha-1}) \end{aligned} \tag{3.8}$$

for $-1 < \alpha \leq -1/2$, ε arbitrary and $n \to \infty$.

Now, to calculate T_3 we use the first order given in (3.2) which holds uniformally in $[\delta, \pi - \delta^1], \delta > 0 \wedge \delta^1 > 0$ but fixed and then apply the Riemann-Lebesgue lemma i.e.

$$\begin{aligned} T_3 &= O(n^{-1/2}) \int_{\delta}^{\pi-\delta^1} F(\varphi)\cos(n\varphi + \gamma)\, d + O(n^{-3/2}) \\ &= o(n^{-1/2}), (n \to \infty) \\ &= o(n^{-\alpha-1}),\ (-1 < \alpha \leq -1/2) \end{aligned} \tag{3.9}$$

To counter the orders of T_4 and T_5 we run through the situations given in Theorems 1, 2, 3 and 4. According to Theorem 1, we have $-1 < \beta \leq -1/2$ only. Thus using the estimate of $P_n^{(\alpha,\beta)}(\cos\varphi)$ from (3.1), we get

$$\begin{aligned} T_5 &= O(n^{\beta}) \int_{\pi-c/n}^{\pi} |F(\varphi)| d\varphi \\ &= o(n^{\beta}) = o(n^{-\alpha-1}) \end{aligned} \tag{3.10}$$

for $f \in L_1^{(\alpha,\beta)}$, $\wedge\, n \to \infty$ and $\{-1 < \alpha \leq -1/2, -1 < \beta \leq -1/2\} \Rightarrow \alpha + \beta + 1 \leq 0$. Choosing δ^1 arbitraily small,

$$\begin{aligned} T_4 &= O(n^{-1/2}) \int_{\pi-\delta^1}^{\pi-c/n} (\cos\varphi)^{-\beta-1/2} |F(\varphi)| d\varphi \\ &= O(n^{-1/2+\beta+1/2}) \int_{\pi-\delta^1}^{\pi-c/n} |F(\varphi)| d\varphi = o(n^{\beta}) \\ &= o(n^{-\alpha-1}) \end{aligned} \tag{3.11}$$

as in T_5 of (3.10) by using the orders of Jacobi polynomials valid in $[\pi - c/n, \pi - \delta^1]$. In the situation of Theorem 2, we have exactly the same calculations, only keeping in mind that in case $\beta > -1/2$ we have to suppose $\alpha + \beta + 1 \leq 0$. In Theorems 3 and 4 we use the 'antipole' conditions (2.6) and (2.5) respectively. Thus for Theorem 3,

$$\begin{aligned} T_4 &= O(n^{-1/2}) \int_{\pi-\delta^1}^{\pi-c/n} (\cos\varphi)^{-\beta-1/2} |F(\varphi)| d\varphi \\ &= O(n^{-1/2}) \int_{c/n}^{\delta^1} (\sin\varphi/2)^{\beta+1/2} |f(-\cos\varphi)| d\varphi \\ &= o(n^{-1/2-\alpha-1/2}) = o(n^{-\alpha-1}) \end{aligned} \tag{3.12}$$

by the antipole condition (2.6) when $\beta > -1/2$ and δ^1 chosen arbirarily small positive real. Moreover,

$$\begin{aligned} T_5 &= O(n^{\beta}) \int_{\pi-c/n}^{\pi} |F(\varphi)| d\varphi \\ &= O(n^{\beta}) \int_0^{c/n} (\sin\varphi/2)^{2\beta+1} |f(-\cos\varphi)| d\varphi \\ &= O(n^{-1/2}) \int_0^{c/n} \varphi^{\beta+1/2} |f(-\cos\varphi)| d\varphi \\ &= o(n^{-1/2} n^{-\alpha-1/2}) = o(n^{-\alpha-1}) \end{aligned} \tag{3.13}$$

by condition (2.6) as $n \to \infty$. Similarly, in Theorem 4 we have $\beta > -1$ and condition (2.5) so that

$$\begin{aligned} T_5 &= O(n^{\beta}) \int_0^{c/n} (\sin\varphi/2)^{2\beta+1} |f(-\cos\varphi)| d\varphi \\ &= O(n^{-1}) \int_0^{c/n} \varphi^{\beta} |f(-\cos\varphi)| d\varphi \\ &= o(n^{-\alpha-1}) \end{aligned} \tag{3.14}$$

as $n \to \infty$. Writing

$$G_2(h) = \int_0^h \varphi^{\beta} |f(-\cos\varphi)| d\varphi$$

so that $\forall \varepsilon^1 > 0, \exists \delta^1 : c/n < h < \delta^1 \Rightarrow |G_2(h)| < \varepsilon^1 h^{\alpha}$ by the meaning of the condition (2.5).

Hence

$$\begin{aligned} T_4 &= O(n^{-1/2}) \int_{\pi-\delta^1}^{\pi-c/n} (\cos\varphi/2)^{-\beta-1/2} |F(\varphi)| d\varphi \\ &= O(n^{-1/2}) \int_{c/n}^{\delta^1} \varphi^{1/2} \varphi^{\beta} |f(-\cos\varphi)| d\varphi \\ &= O(n^{-1/2}) \left\{ \left[\varphi^{1/2} G_2(\varphi)\right]_{c/n}^{\delta^1} - 1/2 \int_{c/n}^{\delta^1} \varphi^{-1/2} G_2(\varphi) d\varphi \right\} \\ &= O(n^{-1/2}) \left\{ \left[\varphi^{1/2} \varepsilon^1 \varphi^{\alpha}\right]_{c/n}^{\delta^1} - 1/2 \int_{c/n}^{\delta^1} \varphi^{-1/2} \varepsilon^1 \varphi^{\alpha} d\varphi \right\} \\ &= o(n^{-\alpha-1}) \end{aligned} \tag{3.15}$$

by the condition (2.5) for $\beta > -1 \wedge n \to \infty$, noting that $-1 < \alpha \leq -1/2$ so that $\alpha + 1/2 \leq 0$. This completes the proof of the Lemma 3.5.

Proof of the Theorems 1, 2, 3 and 4.

Let $S_n(f,1)$ denote the nth partial sum of the Jacobi series at $x = \cos\theta = +1$

$$S_n(f,1) = \sum_{i=0}^{n} a_i P_i^{(\alpha,\beta)}(1)$$

then,

$$\int_0^{\pi} \frac{2^{-\alpha-\beta-1}\Gamma(n+\alpha+\beta+1)}{\Gamma(\alpha+1)\Gamma(n+\beta+1)} 2^{\alpha+\beta+1}$$

$$(\sin\varphi/2)^{2\alpha+1}(\cos\varphi 2)^{2\beta+1} f(\cos\varphi)P_n^{(\alpha+1,\beta)}(\cos\varphi)d\varphi.$$

Thus for any absolute constant A, by the orthogonality of Jacobi polynomials, we have

$$S_n(f,1) - A = \int_0^{\pi} h_n^{(\alpha+1,\beta)} P_n^{(\alpha+1,\beta)}(\cos\varphi)P_n^{(\alpha+1,\beta)}(1)A_n F(\varphi)d\varphi \quad (4.1)$$

by Lemmas 3.3 and 3.4. So

$$|S_n(f,1) - A| = \left|h_n^{(\alpha+1,\beta)} P_n^{(\alpha+1,\beta)}(1)A_n\right| \left|\int_0^{\pi} F(\varphi)P_n^{(\alpha+1,\beta)}(\cos\varphi)d\varphi\right|$$

$$= O(n^{\alpha+1})o(n^{-\alpha-1}) = o(1) \quad (4.2)$$

by Lemma 3.5. This completes the proof Theorems 1, 2, 3 and 4.

Conclusion

In conclusion, it can be seen that signals having countable number of discontinuities of first kind are associated with a Jacobi series which is convergent at the end points called Poles$x = \pm 1$. This indicates that a discontinuous signal can be approximated by uniformly convergent Jacobi series and waveletsso that a contaminated voice signal or corrupted data can be repaired with due techniques and many more other applications.

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